HARDY'S INEQUALITY IN A LIMITING CASE ON GENERAL BOUNDED DOMAINS

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Abstract. In this talk, we study Hardy's inequality in a limiting case:

$$\int_\Omega |\nabla u|^N dx \geq C_N(\Omega) \int_\Omega \frac{|u(x)|^N}{|x|^N \left(\log \frac{R}{|x|}\right)^N} dx$$

for functions $u \in W_0^{1,N}(\Omega)$, where Ω is a bounded domain in $\mathbb{R}^N$ with $R = \sup_{x \in \Omega} |x|$. We study the attainability of the best constant $C_N(\Omega)$ in several cases. We provide sufficient conditions that assure $C_N(\Omega) > C_N(B_R)$ and $C_N(\Omega)$ is attained, here B_R is the N-dimensional ball with center the origin and radius R. Also we provide an example of $\Omega \subset \mathbb{R}^2$ such that $C_2(\Omega) > C_2(B_R) = 1/4$ and $C_2(\Omega)$ is not attained. This talk is based on a joint work with Jaeyoung Byeon (KAIST).

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