On Higher Dimensional Interlacing Fibonacci Sequences, Continued Fractions and Chebyshev Polynomials

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Abstract. We study higher-dimensional interlacing Fibonacci sequences and their corresponding multi-dimensional continued fractions, generated via both Chebyshev type functions and m-dimensional recurrence relations. For each integer m, there exist both rational and integer versions of these sequences, where the underlying p-adic structure of the rational sequence enables the integer sequence to be recovered. In particular, for the positive index sequences, one can clear fractions if one know the number of prime divisors of $2m + 1$; in the negative index case the "excess" prime factors can be removed using Weisman's congruence. When $2m + 1$ is a prime these two processes come into alignment.

From either the rational or the integer sequences we can construct a continued fraction vector in $\mathbb{Q}^m$, which converges to an irrational algebraic point in $\mathbb{R}^m$. The sequence terms can be expressed as simple recurrences, trigonometric sums, binomial polynomials and as sums over ratios of powers of the diagonals of the regular unit n-gon. These sequences also exhibit a "rainbow type" quality, corresponding to the Fleck numbers at negative indices and the m-dimensional Fibonacci numbers at positive indices.

It is shown that the families of orthogonal generating polynomials defining the recurrence relations employed, are divisible by the minimal polynomials of certain algebraic numbers, and the three-term recurrences and differential equations for these polynomials are derived. Further results relating to the Christoffel-Darboux formula, Rodrigues' formula and raising and lowering operators are also discussed Furthermore, it is shown that the Mellin transforms of these polynomials satisfy a functional equation of the form $p_n(s) = \pm p_n(1 - s)$, and have zeros only on the critical line Re $s = 1/2$.

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