On the numerical relaxation of single-slip plasticity in finite strains

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Abstract. The modeling of the elastoplastic behavior of single crystals with infinite latent hardening leads to a nonconvex energy density, whose minimization produces fine structures. The effective macroscopic behaviour can be characterised by means of the quasiconvex envelope of the energy density; unfortunately a closed form expression for the latter is known only in few very simplified cases. One is therefore lead to a new computational challenge, namely numerical relaxation. This is faced with huge numerical difficulties since it involves the minimization of a nonconvex function with clusters of local minima.

With the objective of gaining better insight in the type of microstructure that can develop, and in the type of numerical minimization algorithm that can be used for the relaxation, we study a simplified model problem in two-dimensional, geometrically nonlinear plasticity, with a single slip system and a linear hardening law. A different analysis of the relaxation of the same model was previously given in [1, 4].

By constraining the elastic part of the deformation to be a rotation, we consider a first example where the dissipation contribution to the incremental energy is neglected, and a second one where the plastic free energy is neglected. For both cases, the quasiconvexification of the energy density can be determined in closed form [2, 3].

More refined models are then obtained by assuming the microstructure to have the form of a laminate of second order which is supported either on rigid-plastic deformations, or on purely elastic ones, or on a mixture of purely elastic and plastic ones. In all these cases the relaxation can be reduced to the minimization of a function of only one variable.

We use the above results for the numerical minimization of the full energy density, including dissipation, and removing the kinematic constraint. We then assess the precision of our relaxation by determining at each macroscopic strain a polyaffine function which coincides with the unrelaxed energy on the support of the laminate and checking that it is below the condensed energy, up to a very small error.

We conclude with some numerical examples and comparisons with the literature [1, 4, 5].

References

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